THE NATURE AND FOUNDATION OF LOGIC SYSTEMS:

A RELIABALIST PERSPECTIVE



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PREFACE

I am writing this paper as an undergraduate senior attending California State University, Fresno, for an independent study course in Philosophy, and under the advisement of Dr. Otavio Bueno. My goal for the independent study course was to gain a high level understanding of logics or logic systems. I read three main texts as research for this paper: 1) <u>Philosophy of Logic</u> by Willard V. Quine, 2) <u>Thinking About Logic</u>, by Stephen Read, and 3) an appendix to Formal Methods and the Certification of Critical <u>Systems</u> by John Rushby. The first two texts are standard introductions to the Philosophy of Logic; the third text (or appendix therefrom) was a rapid exposition of various logics including traditional logics, constructivist logics, free logics, and programming logics.

The number and complexity of the issues in the Philosophy of Logic kept me from developing any more than a shallow understanding of fundamental properties of logic systems. However, I applied what little understanding I had to describing the nature and foundation of logic systems in general. No doubt, my descriptions and arguments are *not* beyond reasonable objection, but my description and arguments are intelligent enough to reflect some promise in becoming, with adequate modification and further support, a satisfactory view. I hope you enjoy what follows.

INTRODUCTION

Logic systems such as Propositional logic and First Order logic are systems used for inquiry and in these systems we have utmost confidence. What is the source of this confidence? Do we have good reason to believe that we have access to the truth, enough to say that we can tell merely by introspection that these systems are truth-preserving? Even if we believe we have such access, what convinces us that we have such access?

In my paper entitled, <u>Let's be consistent: Incorporating an Argument from the Radical Sceptic into our Epistemology</u>, I adopted and argued for a reliabalist Epistemology. In this paper, I will do likewise. The foundations of logic systems and any systems used for inquiry are based on reliability. Logic systems are different from informal reasoning systems (ex. criteria of adequacy rules) in that they command our utmost confidence. Access to the truth is not a necessary condition for characterizing the existence and application of logic systems, but reliability is.

In section I, I sketch a characterization of logic systems. In my characterization, I apply a reliabalist and instrumentalist perspective. In section II, I discuss the meta-inquiry that is involved in studying or developing logic systems. I argue that engaging in meta-inquiry is rational. In section III, I sketch how a traditional logic system could have been developed within a reliabalist perspective and why some traditional logics have some of

the properties they do. In section IV, I discuss some semantic anomalies or paradoxes that have moved some to discount traditional logics. I argue that if strong semantic analysis were applied to propositions before they are admitted as premises into a logic system, then the paradoxes would not exist.

In section V, I describe a foundation for logic systems appealing to the weakest and fewest concepts possible, so that the resulting view will be strong. I argue that access to the truth is not necessary for developing or applying a logic system. In section VI, I sketch how my characterization of logic systems is consistent and coheres well with contemporary scientific theory. In section VII, I deal with objections against my characterization. Notions such as intentionality, meaning, consequence, and certainty all have suitable content within a reliabalist perspective.

I: CHARACTERIZATION OF LOGIC SYSTEMS

Principles of inquiry are abstractions on instances of successful inquiries with conclusions of any likelihood. Logic is a set of principles of inquiry with conclusions of certainty. Allow me to explain why this definition is satisfactory. The explanation proceeds on a principle of using the weakest concepts wherever possible. The concept of logics my definition proposes is the weakest functioning concept, and hence the strongest possible view.

Within the history of one's experiences, one finds similarities among the processes by which one has engaged in inquiries and formed conclusions. The curious may study these similarities, eventually forming a set of principles of inquiry. However, this is not the only way by which principles can be developed. Inquiry without principles is very laborious. Each step in each case must be considered at length. The efficient may seek a way to reduce the labor of inquiry eventually forming a set of principles of inquiry. When the principles are applied, they are to form the same conclusion with the same confidence as one would have if the inquiry occurred without the principles. Individuals in agreement on conclusions and likelihood would see the benefit of the principles and the most successful systems of principles would be adopted.

Some inquiries, that is some processes towards a conclusion, afford one a degree of confidence so high no higher confidence can be imagined. Such conclusions are said to

be certain. These are a special case of inquiries for multiple reasons. Some individuals are uncomfortable with conclusions that are uncertain. These individuals will want to study this class of processes in hopes of finding principles that can be applied in a way that extends certainty over a wider scope of inquiry. Other individuals interested in mechanizing inquiry will want to understand the relation between confidence in sub-inquiries and confidence in the overall inquiries. The goal of the study of this class of processes that afford utmost confidence, will be to find principles that can be applied to form conclusions that are certain. These systems of principles are systems of logic.

This characterization explains the nature of our systems of logic. One of the salient properties of logic in this characterization is that logic is developed as a simplification of inquiry. There are numerous possible logic systems that can be developed and used, but the most popular are those that are simple and easily applied. Many others of our beliefs can be explained under this characterization such as why there are multiple logic systems and such as how it is that logic systems have evolved after finding inconsistency in them. Logic is a system developed to automate the inquiry process. Whenever a developed system runs afoul, we need only return to our natural inquiry process to solve the problem. This is what we do until we have the most efficient and well behaved system imaginable.

Some may object to this characterization on grounds that it relativizes logic systems. Traditionally, logic has been seen as a system with a truth preservation property. Under my characterization, logic need only preserve an individual's inquiry biases. A logic system can be developed for each bias and each individual can have a different bias. Therefore, this characterization does relativize logic. This objection is unfounded however, because reliable bias (or organization) and not truth is a stronger foundation for logic systems because it is satisfactorily explanatory and involves weaker claims.

II: META-INQUIRY OF LOGIC SYSTEMS

An inquiry such as this one into the nature of logic must be perceived of as useful before obtaining interest. Inquiry influences behavior, and behavior influences the course of one's life. If one's life is of interest, so ought inquiry. But what of the *study* of inquiry? Inquiry is *itself* important, and one finds that some of one's conclusions are later abandoned or reversed by one. One finds that one's conclusions were mistaken or unreliable. But what is it that makes some conclusions reliable and others unreliable? This problem within one's own inquiry mechanism prompts one to *study* inquiry itself in hopes of modifying ones inquiry mechanism to produce more consistently reliable conclusions. Furthermore and as has already been mentioned, inquiry and conclusion forming is laborious. Studying inquiry may allow us to develop a system of inquiry which reduces the labor. The more efficient and more reliable one is, the better chance one has at survival in a harsh world.

But is this meta-inquiry possible? Meta-inquiry presupposes some access to the inquiry mechanism; it presupposes some feedback mechanism. One needs to be able to determine what one's mechanism is doing (i.e. what steps is it carrying out and in what

order) and one needs to be able to modify it according to the conclusions of the metainquiry. If one does not satisfy these conditions, the meta-inquiry will be unreliable or useless. There is danger in engaging in a meta-inquiry without satisfying pre-conditions. One can negatively alter one's otherwise mostly reliable inquiry mechanism, if one had faulty access. However, the same can be said for engaging in any behavior including the behavior of doing nothing. For survival in a harsh world, one may need to take the risk and attempt the optimizations subscribed by meta-inquiries. The world that our natural mechanism affords us is a harsh one, one that will require developing logic systems for our survival.

These are the considerations one should have before engaging in meta-inquiry and applying its conclusions. In order to be cautious one ought to test the conclusions of a logical system constructed from the meta-inquiry with the conclusions from one's natural mechanism. This places the faith first in one's natural abilities, and second in the developed system. That is, a logic system is developed in order to improve one's behavior in the world. Whether it is accepted depends on its ability to do just that and this is determined according to one's natural system; for having the developed system determine it would provide no suitable test. A system could be easily developed to only allow conclusions that it was improving one's behavior.

III: FRUITS OF META-INQUIRY

A logic system may be developed entirely independently of the steps its natural mechanism takes in forming conclusions. The natural mechanism need only affirm that the logic system is useful in its application. However, presumed access to the steps one's mechanism takes may be taken as a guide for developing a system. The meta-inquiry may determine that some objects preclude other objects and include yet others. For example, "lake" precludes "dry land" yet includes "water". One principle, then, is based on the semantics of objects or simulations of objects. However, this principle is really a multitude of individual principles, multiple principles for each object. Semantic analysis is not *amenable* to simple mechanization or simplification. The rules afforded by semantic analysis would be so numerous and are so quickly determined anyway, that making them principles in a logic system that is intended to be applied by a human.

However, semantic analysis does provide for a notion of implication (ex. if there is a lake, then there is no dry land). A notion of implication can also be found in the close connection between events (ex. if I eat, then I am satisfied). A meta-inquiry locates the similarities between instances of inquiry, and implication between beliefs is one of the frequently observed similarities. Other traditional notions of logic can be obtained through this meta-inquiry. Disjunction can be obtained through the implication of multiple events from a given event (ex. if I go hunting, I either find a rabbit or a bird). Negation is frequently observed as well (ex. I am not satisfied). Meta-inquiry observes that in some cases relations between disjunction, negation, implication, and conjunction

are all that is needed to determine the eventual conclusion of one.¹ Because of this, a suitable enumeration of these principles forms a logic system.

The deeper the inquiry goes, the more similarities are given significance, and the more principles are added to the logic system. Existential and Universal quantifiers, collections of objects, and types are all abstractions on what is frequently found within our natural inquiries. However, if we create logic systems that do not require semantic analysis at each step, then problems arise in traditional logic systems. That is, a natural inquiry mechanism does not allow statements which are problematic such as the liar paradox² or paradoxes based on statements about fictional objects³. These statements hardly arise within natural reasoning and if they do, they are quickly and reliably dealt with by a human. However, considering them in relation to logic systems (with an emphasis on traditional logic systems) tests one's characterization of logic systems. For this reason, both types of paradoxes will be considered.

IV: SEMANTIC ANOMALIES

One problem is that grammar allows us to make statements which appear to be informative, but are really uninformative. For a computer, semantic analysis is very difficult; so most computers engaging in theorem proving or other automated reasoning, will only analyze the syntax or grammar of statements. But numerous sentences can have questionable meaning, yet be grammatically correct. Some examples have already been

¹ This is similar to determining the causal relations between external events.

² For example, "This statement is false."
³ For example, "Santa Claus loves chicken."

mentioned. Two others are: "This statement is true" and "Superman had a pet bird". The first appears to have no suitable content; what information does it provide? The second regards a property of a fictional character; but are fictional characters complete objects which instantiate either the truth or falsity of each possible property?⁴ Being a grammatical statement does not guarantee being informative or being useful in reasoning. How should logic systems handle such cases?

Traditionally, logic is a set of principles from which some information can be translated or transformed into other information. Semantic analysis falls outside of logic, because principles of semantic analysis are too numerous. But are not the stated problems semantic in nature? A semantic analysis of the "liar" and its variants shows that such statements contain no information. Other more complex statements can be seen to contain no information as well (ex. if the antecedent of this conditional is true, then the consequent of this conditional is true). Since these statements contain no information, they are not properly introduced into a logic system. This does not preclude statements such as P or not P from being introduced. Such statements signify information which is inherent in the logical system, that a statement is either true or false, and that meaningless statements are not accepted in the system. The reiteration of informative statements is different from the iteration of uninformative ones.

So far, the problem is thinking that every grammatical statement contains some information. But again, what of statements about fictional or hypothetical objects? Hypothetical objects are used extensively in reasoning. Our conclusions are frequently

⁴ Another case is descriptive statements that fail to denote.

based on simulations of hypothetical objects. One tries to give one's hypothetical objects all the relevant properties necessary for the simulation, but what if some are missing? Statements such as "Mickey Mouse had ears" are true; statements such as "Mickey Mouse was a cat" are false. But what of statements such as, "Mickey Mouse ate pizza every Sunday night"? They appear to contain information, yet there is no way to answer this question. Mickey Mouse does not have the relevant properties for answering.

Such statements are analogous to another class of sentences. These sentences are problematic as well. "The sky was happy" or "A square ate broccoli" are grammatical statements, apparently informative, yet are likewise problematic. One reason both classes of sentences are problematic are that making them false, appears to imply that these properties are relevant properties of the objects. It is not that Mickey Mouse *did not* eat pizza every Sunday night; it is that Mickey Mouse did not exist every Sunday night. Nothing can be said about what he did or did not do *every* Sunday night. It is not that the sky was not happy (i.e. unhappy), but that a sky cannot be happy nor unhappy; such properties are not relevant for the object. They do not provide information about an entity.

One problem here is thinking something follows from an object not having a property that does not really follow. If the statement about the sun being happy is false, the conclusion that the sun is unhappy does not follow. If a statement about there being only a few marbles in my hand is false, the conclusion that there is many does not follow. Jumping to conclusions is an epidemic problem for humans and is supported by the countless explanations and philosophies that have been believed throughout the ages. Some objects do not have the property of happiness, not because they are unhappy, but because they are not objects which are capable of happiness nor unhappiness.

The solutions to problematic statements sketched here push the problem away from logic systems and onto semantic analysis. Strong semantic analysis must take place upon the statements used as premises for introduction into the logic system. Within the premises must not be any uninformative statements nor unfounded conditionals. If reasoning is to be automated such as by a computer to the degree that a computer creates new statements for testing, then such strong semantic analysis must be built into the computer to keep it from introducing uninformative statements. However, this does not imply there is a problem with a logic system or that logic systems ought to deal with these semantic anomalies. Traditional logic is properly designed to deal with information and not semantic anomalies for dealing with semantic anomalies would multiply the number of rules of the system and make it impractical for human use.

V: FOUNDATIONS FOR LOGIC SYSTEMS

Logic systems are formal systems, and formal systems are systems. Systems need only work in order to be used. Some philosophers have described humans as having access to the truth; a system that provides access or preserves access to the truth would work, but truth is not necessary for a working system. A system without access to the truth may work better. For the purposes of this paper, a system defines or determines how one comes to conclusions or forms beliefs. Much of the every day reasoning is part of an informal system. Humans form beliefs not through the explicit application of rules, but through the natural application of themselves. Logic is a formal system, requiring that its inputs and the transformations on its inputs be well defined and explicitly applied.

What causes a man to believe one thing over another? Is it some intuitive access to the truth, or is it the ability the belief gives one to make predictions? A stable state of mind is what a purposeful entity seeks. From this we can see that logic is happened upon as reliable system, a system that helps one to make reliable predictions, predictions that allow a more stable state of mind. At bottom must be some access not to the truth, but to information about one being hurt or not being hurt.

This access is all that is necessary to test systems, and judge if they are reliable. Access to hurt is not properly thought of as access to the truth for if that were so, then an entity must have the concept of hurt as a relation between itself and states of being. Access to hurt need not be so rich; access to hurt only need be a mechanism that prompts the system in use by one to change. In this way, a reliable system of logic may be obtained.

VI: COHERENCE WITH ONE'S BELIEF SYSTEM

The foundation for a characterization of a logic system based on reliability is difficult to give. Only the most fundamental principles in one's belief forming mechanism should be used in developing and articulating any notion such as logic. This is what I sought to do in the previous section. However, a test for a characterization (and one's less basic belief forming operations) is coherence with one's other beliefs. The foundation of logic is that it works and that its operations are applied with utmost confidence. But how does a mind: 1) engage in the semantic analysis required for introducing some premises into the system? 2) form premises for the system to act upon? and 3) apply the rules of the logic system to the premises?

The simplest explanation presupposes an entity of parts deterministically influenced by one another and with no outside influence. The view that humans are such entities is compatible with modern day science. The universe is deterministic at the atomic level. And an explanation of how a human works need only draw from particles at the atomic or higher levels. Computers though not yet able to develop and apply a logic system on their own, have been successful in performing complex operations of a similar nature. Bundles of neurons in the brain provide for a means of both spatial and temporal distinctions of messages and operations.

This ability to pass distinguishable messages as well as perform operations or transform messages coupled together with an ability to store messages through neuron training is all that is necessary to run a useful and reliable logic system. Content of messages is internally defined, and the ability to transform these messages (which is a capability of a neural net) is the work of a logic system. This is by no means a conclusive explanation of how a logic system can be carried out by a purely mechanistic entity, but it does highlight that a mechanistic entity such as a human does *appear* to have all the necessary properties for a logic system to be realized and in a reliable way.

VII: OBJECTIONS

Tying logic to reliabalism and not to truth, allows for multiple incompatible systems of logic. In the harshest world possible, only one logic system will be reliable, but it still is unnecessary for it to be true. That one lives in the harshest world possible is likely not the case, so there probably are multiple reliable and incompatible systems of logic. I have already defended my characterization against claims that the resultant relativism of a logic system is unsatisfactory.

An objection may be made that consequence only makes sense in true systems and does not make sense in reliable ones. Only if P really does imply Q does P imply Q. The problem with this objection is that consequence as the objector conceives it, is not part of a reliabilist logic. Implication need only be a reliable transformation; it need not be based on true consequence. Learning that P implies Q is a result of training and not a result of access to a consequence relation.

What is certainty if consequence relations are not based on truth? Certainty can be characterized in terms of behavior of the *mental* states of a mechanism. External

behavior is based on information, but so is internal behavior, the internal behavior of the mind. Certain information results in singular internal and external behavior. Uncertain information does not result in singular internal behavior. Uncertain information influences one to consider behavior which will handle the uncertainty. Instead of running full speed and in a perfect line towards a tree containing fruit, one may walk and walk in very small figure eights to make use of its physical exertion. Uncertain information causes the mind to consider multiple scenarios, branching out and multiplying with every uncertain inference. Inferences with certainty do not branch.

Reliabalism provides an adequate foundation for logic systems. Notions of intentionality, meaning, consequence, and certainty can all be given a consistent and coherent account within reliabalism. The resulting relativism of logic systems and belief forming mechanisms in general is not unsatisfactory, as long as the logic systems and belief forming mechanisms do their jobs which is an implicit assumption within reliabalism. Further, the assumptions of the reliabalist view applied in this paper are weaker than the assumptions of views based on access or correspondence to truth.

SUMMARY

A logic system is a set of rules or principles to explicitly follow when engaging in an inquiry. The system transforms *some* information into some other information with certainty. The goal when developing a logic system is that it will be reliable. Testing the reliability of a logic system (at bottom) requires some access to hurt. Access to hurt is certain because if there were not hurt, then any system would be reliable. The resulting relativism of logic systems given this characterization does not support rejection of it. The assumption that one has access to the truth is a stronger claim than one has access to hurt. A reliabalist foundation requires weaker claims; hence, is a stronger position.

The subject discussed and the claims made require a more extensive treatment than was here given them. The sometimes vague statements should be made more detailed. The sometimes questionable moves should be further supported and explained. However, the weakness of the correspondence theorist or representationalist position should have been illuminated to some degree. An account of logic systems as well as all cognition will one day be given a purely mechanistic and reliable explanation. The only conceptual entity that I can see causing a mechanist trouble will be qualia, and qualia seems to be a problem for everyone.

APPENDIX A: 2001 CAP CONFERENCE AT CMU

This summer's Computing and Philosophy Conference at Carnegie Mellon University contained some talks exhibiting some interesting research. The conference lasted two and one half days, but very few speakers presented anything interesting for someone seeking answers to problems in Philosophy. Many talks were given on problems in instruction of Philosophy or problems on publishing philosophical works. Any problem that was related at all to computers and that a philosopher or Philosophy teacher might be interested in was fair game at the conference. As a result, very few "meaty" talks from a philosophical perspective were given.

The meatiest of the talks was presented by Dr. Patrick Grim of the State University of New York, Stony Brook. It was entitled, "The Emergence of Communication: Some Models of Meaning." Dr. Grim, with a team of others, wrote a computer program to simulate a primitive environment in which communication may have emerged. The environment consisted of 4,096 artificially intelligent entities spatially located on a square grid of 4,096 positions; each position contained an entity, and an entity was not allowed to move. An entity was capable of four actions: opening its mouth, hiding, and making two distinct sounds. On the grid roamed a food source (i.e. an edible animal) and also a source of hurt (i.e. a dangerous animal). An entity must have its mouth open when

the food source landed on its square in order to make use of it. An entity must be hiding in order to not be hurt by the predator.

The best possible state of an entity would be to make a particular sound every time food landed on its square in order to notify its neighbor to open its mouth, and to make a particular sound every time a predator landed on its square in order to notify its neighbor to hide. This was one of the two best strategies possible even for an egoist; the other strategy was symmetric to it, using opposite sounds for opposite notifications. The initial states of the entities would be randomly distributed, and the program would be run. Every so many rounds, entities who were not eating and being hurt by predators learned from the most successful of their neighbors about how they were acting. Eventually, almost all of the entities on the grid were using one of the best strategies.

This presentation appeared to bother many of the philosophers in the audience. Many of them that spoke did not seem willing to accept that meaning and intentionality is only a mechanistic process that gets the job done. I do believe that meaning and intentionality are concepts that traditionally are too mentalistic. However, there is something to be said about how a mind, even one that is purely physical and mechanical, is organized in a way that it appears to be thinking in semantic units such as concepts. The infinite number of sentences that can be created from a finite vocabulary appear to move even a hard nosed mechanistic to stipulate further the requirements for meaning and intentionality than what were exhibited by the artificially intelligent entities in the presentation.

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